since they have a convexity opposing the normal displacements to the middle surface caused by the thermal loads and a continuous change in the principal radius of curvature along the meridian.

NOTATION

T, temperature; x^1, x^3, coordinate system axes coupled normally to the shell middle surface; q^1, q^3, contravariant components of the thermal flux density vector in the x^1 and x^3 directions, respectively; g, determinant of the metric form; c, specific heat; ρ, density; T_0, initial temperature; Θ_+, air temperature; Θ_-, temperature of the medium washing over the shell at the boundary Γ_-; α_+, α_2, α_-, heat elimination coefficients to the boundaries Γ_+, Γ_2, Γ_-; Δt, time period between the beginning of slag pouring and the time of its termination.

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SOLUTION OF THE INVERSE COEFFICIENT PROBLEM OF HEAT CONDUCTION

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UDC 536.24.02

An effective algorithm is developed for solving the inverse coefficient problem of heat conduction. Methods are proposed for improving the nonuniform convergence of the solution at the domain boundaries.

In the past decade the problem of determining the thermophysical characteristics of materials in nonstationary heating regimes has attracted the considerable attention of researchers. This is related primarily to the impossibility of determining the mentioned characteristics by classical methods in a number of cases of practical importance, for example, for coking materials, as well as to the possibilities being disclosed here of a substantial rise in the productivity and simplification of experimental investigations. However, serious difficulties associated with the necessity to solve inverse coefficient problems of heat conduction occur here. The development of methods to solve these problems was studied in [1-10].

In conformity with [1, 2], and using the method proposed in [11] to calculate the gradient of the functional of the quadratic residual of experimental and computed temperatures, an effective algorithm to solve the inverse coefficient problem of heat conduction is given in this investigation which would assure good accuracy in determining the heat conduction coefficient $\lambda(T)$ and the specific heat $c(T)$.

Let us consider the solution of the inverse problem for an infinite plate. In order not to limit the possibilities of the experiment we consider that boundary conditions of the first, second, and third kinds can be given on the plate surfaces. The temperature distribution in the plate will hence be described by the equation

$$c(T)\rho\,\frac{\partial T}{\partial t}=\frac{\partial T}{\partial x}\left(\lambda(T)\,\frac{\partial T}{\partial x}\right),\quad 0<x<\delta,\ 0<t\leqslant t_e \tag{1}$$

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with the boundary conditions

$$r_1\lambda(T)\left.\frac{\partial T}{\partial x}\right|_{x=0} = \alpha(t)T(0,\ t) - \varphi(t), \qquad 0 < t \leqslant t_e \tag{2}$$
$$r_2\lambda(T)\left.\frac{\partial T}{\partial x}\right|_{x=\delta} = -\beta(t)T(\delta,\ t) + \psi(t)$$

and the initial distribution

$$T(x,\ 0) = T_0(x),\ 0 \leqslant x \leqslant \delta. \tag{3}$$

The coefficients r_1, r_2 that take on the values 0 or 1, and the functions $\alpha(t)$, $\beta(t)$, $\varphi(t)$, $\psi(t)$ assure the possibility of giving arbitrary boundary conditions of the types mentioned on the plate surfaces.

To determine the unknown thermophysical characteristics, let there be additional information in the form of experimental curves of the temperature change in K at points of the plate

$$T^e(x_k,\ t) = T_k^e(t),\ k = 1,\ 2,\ \ldots,\ K. \tag{4}$$

From the viewpoint of the mathematical formulation of the inverse problem it is sufficient to give the additional information at one point. However, because of the inevitable errors in determining the position of a point, in the temperature measurement, and because of the substantial dependence of the solution of inverse problems on errors in the initial data, it is expedient to use the greatest possible quantity of information in order to assure a statistical diminution in the influence of these errors on the solution being obtained.

It should also be noted that if (1) and the boundary conditions (2) are homogeneous in λ and c, then the dependences $\lambda(T)$ and c(T) can be determined only to the accuracy of a constant factor. To obtain a single-valued solution in the simultaneous determination of $\lambda(T)$ and c(T), additional conditions are necessary, giving the magnitude of one of the characteristics in at least some one point of the temperature domain under consideration, or inhomogeneous boundary conditions in λ and c on one of the surfaces.

Assuming that the mentioned single-valuedness requirements are satisfied, the problem of determining $\lambda(T)$ and c(T) can be formulated from the condition of the best approximation of all available sets of experimental data by the solution of problem (1)-(3). Since the experimental values of the temperature contain random errors of measurement and decoding, it is expedient to take the magnitude of the root-mean-square deviation of the measured value from the computed values of the temperature as a measure of the approximation.

We then obtain for $\lambda(T)$ and c(T)

$$S = \frac{1}{\sum\limits_{k=1}^{K}\int\limits_0^{t_e}\mu_k dt}\sum_{k=1}^{K}\int\limits_0^{t_e}\mu_k[T(x_k,\ t) - T_k^e(t)]^2\,dt = \min, \tag{5}$$

where $\mu_k = \mu(x_k,\ t)$ are given weight factors characterizing the confidence and informativeness of the experimental data.

The functions $\lambda(T)$ and c(T) that communicate the minimum to the functional (5) will indeed be the desired temperature dependences of the thermophysical characteristics of the temperature under investigation. The finite-dimensional representation of the functions being determined

$$\lambda(T) = \lambda(\lambda_1,\ \lambda_2,\ \ldots,\ \lambda_N),\ c(T) = c(c_1,\ c_2,\ \ldots,\ c_M) \tag{6}$$

permits reduction of the problem to seeking the minimum of the function $S(\lambda_1,\ \ldots,\ \lambda_N,\ c_1,\ \ldots,\ c_M)$ with respect to the finite (N + M) number of variables, the parameters of the representation taken.

It is desirable to select the form of the finite-dimensional representation in conformity with a priori information about the behavior of the functions being sought. In the general case, there is no such information, and consequently, the selected shape must assure a good approximation of the function for an arbitrary law of variation. A tabular representation of the function with its values determined between the interpolation nodes will satisfy this condition.

To solve the nonlinear boundary-value problem (1)-(3) we use an implicit four-point difference scheme for a quasilinear equation with the order of approximation $O(\tau + h^2)$ and an iteration method of solving the system of difference equations for going from one time layer to another.

On the segments $0 \leqslant x \leqslant \delta$ and $0 \leqslant t \leqslant t_e$ we introduce uniform meshes with the nodes $x_i = (i-1)h$, $i = 1, 2, \ldots, I$, $t_j = j\tau$, $j = 1, 2, \ldots, J$, $h = \delta/(I-1)$, $\tau = t_e/J$. Then the following system of equations will be the difference analog of the problem (1)-(3)

$$r_1 c_{1j}\rho \frac{\overset{s+1}{T}_{1j} - T_{1j-1}}{\tau} = \frac{2}{h}\left(r_1\lambda_{1j}\frac{\overset{s+1}{T}_{2j} - \overset{s+1}{T}_{1j}}{h} - \alpha_j \overset{s+1}{T}_{1j} + \varphi_j\right),$$

$$c_{ij}\rho \frac{\overset{s+1}{T}_{ij} - T_{ij-1}}{\tau} = \frac{1}{h}\left(\lambda_{ij}\frac{\overset{s+1}{T}_{i+1j} - T_{ij}}{h} - \lambda_{i-1j}\frac{\overset{s+1}{T}_{ij} - \overset{s+1}{T}_{i-1j}}{h}\right), \tag{7}$$

$$r_2 c_{Ij}\rho \frac{\overset{s+1}{T}_{Ij} - T_{Ij-1}}{\tau} = \frac{2}{h}\left(-r_2\lambda_{I-1j}\frac{\overset{s+1}{T}_{Ij} - \overset{s+1}{T}_{I-1j}}{h} - \beta_j \overset{s+1}{T}_{Ij} + \psi_j\right),$$

$$i = 2, 3, \ldots, I-1, \quad j = 1, 2, \ldots, J,$$

where

$$c_{ij} = c\left(\frac{\overset{s}{T}_{ij} + T_{ij-1}}{2}\right); \quad \lambda_{ij} = \lambda\left(\frac{\overset{s}{T}_{i+1j} + \overset{s}{T}_{ij}}{2}\right);$$

$$T_{i0} = T_0(x_i); \quad \overset{0}{T}_{ij} = T_{ij-1}.$$

The superposed superscripts denote the number of the iteration at which the quantity noted is evaluated.

The system of difference equations (7) is solved by using an iteration process and the factorization method at each time spacing. For the majority of dependences $\lambda(T)$, $c(T)$ the iteration process converges rapidly, and two to three iterations are sufficient.

Correspondingly, we obtain for the functional (5)

$$S = \frac{1}{\sum_{k=1}^{K}\sum_{l}\mu_{kl}}\sum_{k=1}^{K}\sum_{l}\mu_{kl}(T_{kl} - T^{e}_{kl})^2, \tag{8}$$

where T_{kl} equal the values of the mesh functions T_{ij} at the nodes to which x_k and t_l agree, while they are evaluated by interpolation when there is no such agreement.

To evaluate the gradient of the functional (8) we use the method proposed in [11] for finding the gradient of similar functionals for linear heat conduction problems. Extending this method to the case of the nonlinear problem, we obtain the following system of equations adjoint to (7):

$$-\frac{r_1\rho}{\tau}(c^{-}_{1j+1}b_{1j+1} - c^{+}_{1j}b_{1j}) = \frac{\lambda^{+}_{1j}}{h^2}(b_{2j} - 2r_1 b_{1j}) - \frac{2\alpha_j}{h}b_{1j} + \frac{\partial S}{\partial T_{1j}},$$

$$-\frac{\rho}{\tau}(c^{-}_{2j+1}b_{2j+1} - c^{+}_{2j}b_{2j}) = \frac{\lambda^{+}_{2j}}{h^2}(b_{3j} - b_{2j}) - \frac{\lambda^{-}_{1j}}{h^2}(b_{2j} - 2r_1 b_{1j}) + \frac{\partial S}{\partial T_{2j}},$$

$$-\frac{\rho}{\tau}(c^{-}_{ij+1}b_{ij+1} - c^{+}_{ij}b_{ij}) = \frac{\lambda^{+}_{ij}}{h^2}(b_{i+1j} - b_{ij}) - \frac{\lambda^{-}_{i-1j}}{h^2}(b_{ij} - b_{i-1j}) + \frac{\partial S}{\partial T_{ij}}. \tag{9}$$

$$i = 3, 4, \ldots, I-2,$$

$$-\frac{\rho}{\tau}(c^{-}_{I-1j+1}b_{I-1j+1} - c^{+}_{I-1j}b_{I-1j}) = \frac{\lambda^{+}_{I-1j}}{h^2}(2r_2 b_{Ij} - b_{I-1j})\ \frac{\lambda^{-}_{I-2j}}{h^2}(b_{I-1j} - b_{I-2j}) + \frac{\partial S}{\partial T_{I-1j}},$$

$$-\frac{r_2\rho}{\tau}(c^{-}_{Ij+1}b_{Ij+1} - c^{+}_{Ij}b_{Ij}) = -\frac{\lambda^{-}_{I-1j}}{h^2}(2r_2 b_{Ij} - b_{I-1j}) - \frac{2\beta_j}{h}b_{Ij} + \frac{\partial S}{\partial T_{Ij}},$$

$$j = 0, 1, \ldots, J, \quad b_{iJ+1} = 0, \quad i = 1, 2, \ldots, I,$$

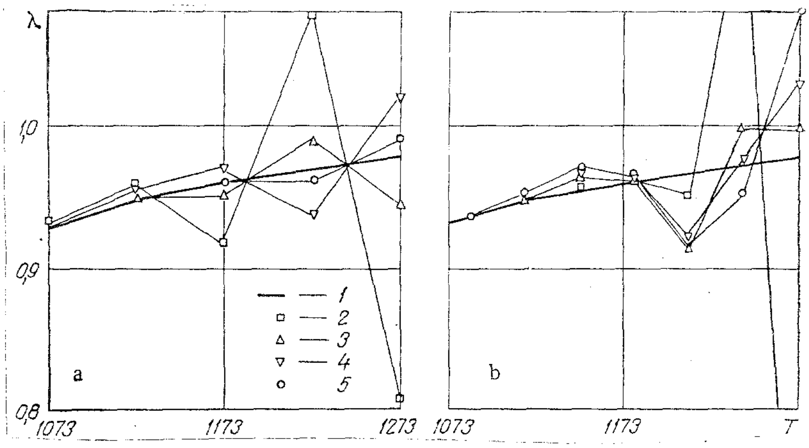

Fig. 1. Convergence of the solution of the domain boundaries: 1) desired dependence $\lambda(T)$; 2) solution; 3) solution for d = 2; 4) d = 3; 5) d is determined at the initial spacing (a); 1) desired dependence $\lambda(T)$; 2) solution; 3) solution for $f'(T_N) = 0$; 4) $f'' = 0$; 5) $f''' = 0$ (b); λ, W/mK; T°K.

where

$$\lambda_{ij}^{+} = \lambda_{ij} + 0.5(T_{ij} - T_{i+1j})\lambda'_{ij}; \quad \lambda_{ij}^{-} = \lambda_{ij} - 0.5(T_{ij} - T_{i+1j})\lambda'_{ij};$$

$$c_{ij}^{+} = c_{ij} + 0.5(T_{ij} - T_{ij-1})c'_{ij}, \quad c_{ij}^{-} = c_{ij} - 0.5(T_{ij} - T_{ij-1})c'_{ij}.$$

Here λ'_{ij} and c'_{ij} are the derivatives with respect to the temperature at the points $0.5(T_{ij} = T_{i+1j})$ and $0.5(T_{ij} + T_{ij-1})$, respectively.

Equations (9) are linear in b_{ij} and can be solved by the factorization method. The components of the gradient are determined in terms of the solution of the adjoint equation by the following expressions:

$$\frac{\partial S}{\partial \lambda_n} = \frac{1}{h^2} \sum_{j=1}^{J} \left\{ \sum_{i=2}^{I-1} b_{ij} \left[\frac{\partial \lambda_{i-1j}}{\partial \lambda_n} T_{i-1j} - \left(\frac{\partial \lambda_{ij}}{\partial \lambda_n} + \frac{\partial \lambda_{i-1j}}{\partial \lambda_n} \right) T_{ij} + \right.\right.$$
$$\left.\left. + \frac{\partial \lambda_{ij}}{\partial \lambda_n} T_{i+1j} \right] - 2r_1 b_{1j} \frac{\partial \lambda_{1j}}{\partial \lambda_n} (T_{1j} - T_{2j}) + 2r_2 b_{Ij} \frac{\partial \lambda_{I-1j}}{\partial \lambda_n} (T_{I-1j} - T_{Ij}) \right\},$$
$$n = 1,\ 2,\ \ldots,\ N, \tag{10}$$

$$\frac{\partial S}{\partial c_m} = \frac{\rho}{\tau} \sum_{j=1}^{J} \left\{ \sum_{i=2}^{I-1} b_{ij} (T_{ij-1} - T_{ij}) \frac{\partial c_{ij}}{\partial c_m} + r_1 b_{1j} (T_{1j-1} - T_{1j}) \frac{\partial c_{1j}}{\partial c_m} + \right.$$
$$\left. + r_2 b_{Ij} (T_{Ij-1} - T_{Ij}) \frac{\partial c_{Ij}}{\partial c_m} \right\}, \quad m = 1,\ 2,\ \ldots,\ M.$$

Therefore, the determination of the gradient of the functional of the quadratic residual (8) reduces to the solution of the system of difference equations (7) of the direct problem, the solution of the adjoint system of equations (9), and the evaluation of the component of the gradient by means of (10). The computation time on an electronic computer here increases by more than twofold as compared to the time to solve the direct problem for an iteration residual equal to 0.01% of the magnitude of the temperature interval.

The presence of such an economical method of determining the gradient permits efficient utilization of different iteration methods of gradient type for minimization of the functional (8). Testing the gradient methods, the method of steepest descent, and the method of conjugate gradients on model problems to determine the temperature dependence of the coefficient of heat conduction showed that the method of conjugate gradients yields the best results. However, it should be noted that uniform convergence of the solution at the domain boundaries is not assured in all cases. The nature of the solution (curve 2) at the upper boundary of

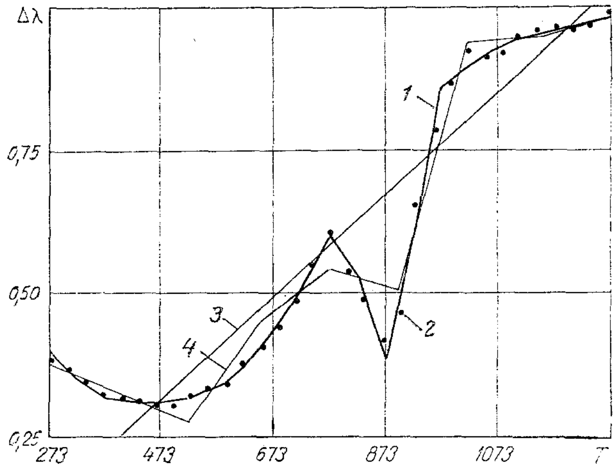

Fig. 2. Restoration of the dependence $\lambda(T)$ by the method of successive minimization: 1) desired dependence; 2) solution; 3) results of a computation for N = 2; 4) for N = 9.

the domain, obtained when using linear interpolation of the function $\lambda(\lambda_1, \lambda_2, \ldots, \lambda_N)$ between nodal points is displayed in Fig. 1a. Utilization of higher-order interpolation (quadratic, cubic, and by the method of smooth execution) does not result in improvement of the results.

As an analysis of the computing process showed, such a behavior of the solution is associated with the fact that the components of the gradient of the functional (8), corresponding to the boundary nodes of the function $\lambda(\lambda_1, \lambda_2, \ldots, \lambda_N)$, are several times less (two and more times for linear interpolation) than the components corresponding to the internal nodes. This latter is due, as before, to the fact that for the boundary nodes the tabulated function $\lambda(\lambda_1, \lambda_2, \ldots, \lambda_N)$ of the quantity $\int \frac{\partial\lambda}{\partial\lambda_n} dT$ is less than for the internal nodes (two times in the case of linear interpolation, and eight times by the method of smooth execution), and is also associated with the nature of the temperature field in the specimen, with the system of initial data and the magnitude of the spacing for the tabulated representation of the function $\lambda(T)$. Consequently, in minimizing the functional (8) the boundary points of the functions being sought $\lambda(\lambda_1, \lambda_2, \ldots, \lambda_N)$ move considerably more slowly to the solution than do the others, which indeed causes distortion of the solution being obtained at the domain boundaries. This is also confirmed by the results of computations for the cases when the boundary components of the gradient are multiplied by the constant d during minimization. Results for d = 2 and 3, respectively, are shown by curves 3 and 4 in Fig. 1a, and by curve 5 for the case when d is determined from the condition of equality of the gradient components at the boundary and nearby internal points at the initial step of the minimization, where the initial approximation should be sufficiently remote from the desired solution so that local singularities would not exert influence on the magnitude of d. Such a definition of the coefficient d takes account of the influence of all the factors mentioned on the boundary component and, as is seen from the results presented, assures a substantial improvement in the solution at the boundaries. This method is certainly of a purely phenomenological nature and cannot always be effective. Consequently, the results being obtained must be checked during its utilization, for instance, by solving for different initial approximations on different sides of the solution being obtained. It should be noted that such a confirmation of the solution is desirable even when using any other method.

Methods of improving the convergence of the solution by introducing constraints on the behavior of the function to be sought at the domain boundaries (first, second, or third derivatives equal to zero) were also considered. The results of computations, presented in Fig. 1b, show that these constraints diminish but do not eliminate distortions of the solution.

The most effective elimination of the nonuniform convergence of the solution was given by utilization of the successive minimization process proposed in [2], which consists of the successive increase in the number of parameters by which the functional is minimized, which corresponds to a 1.5-2 times increase in the number of nodes, starting with two nodes, say,

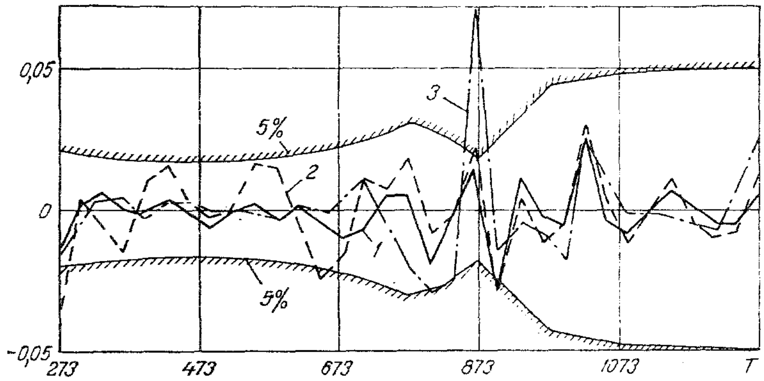

Fig. 3. Errors in restoration of $\lambda(T)$: 1) at exact values of $T^e(X_k, t)$; 2) at rounded off values to 0 and 5°K; 3) for a simultaneous determination of $\lambda(T)$ and $c(T)$. Along the ordinate axis $\Delta\lambda$, W/mK.

under the conditions of this problem. Improvement in the convergence is here achieved by tha fact that the initial minimization in a coarse mesh will assure a good initial approximation for minimization on a finer mesh. Minimization is accomplished at each stage by the method of conjugate gradients by using linear interpolation of the function being sought and by doubling the boundary components of the gradient.

Results of restoring a very complex function $\lambda(T)$ are shown in Fig. 2 (compared to the example presented in Fig. 1, a deep trough in the function is introduced at T = 873°K), from data on the temperature at ten points arranged uniformly over the thickness of the specimen. Presented here are certain intermediate results illustrating the successive minimization process. The good convergence of the solution at both the domain boundaries and at sites of a rapid change in the function should be noted. Errors in the restoration of $\lambda(T)$ when solving the problem at exact values of the temperature (1), at values rounded off to 0 and 5°K (2), as well as the boundary of the 5% error of $\lambda(T)$ and (T) are shown in Fig. 3. The magnitudes of the root-mean=square errors for the cases mentioned are 1.7, 2.7 and 2.9%, respectively, relative to the mean value of λ in the 273-1278°K range.

NOTATION

x, coordinate; T, temperature; t, time; ρ, density; c, specific heat; λ, coefficient of heat conduction; δ, plate thickness; $\alpha(t)$, $\beta(t)$, $\varphi(t)$, $\psi(t)$, functions; r_1, r_2, coefficients; μ_k, weight factors; h, τ, mesh spacings in the space and time coordinates; T^e, experimental values of the temperature.

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PROCEDURE FOR CALCULATING A CONDUCTIVE HEAT EXCHANGER

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In a number of devices a fluid is heated as it flows through a porous wall or a metal wall with a high thermal conductivity which has channels containing the flowing fluid. There naturally arise problems of finding the temperature distribution in the material of such a wall heat exchanger, the temperature change of the flowing fluid, and the amount of heat transferred for given geometric parameters of the wall heat exchanger and given values of the initial temperature and the flow rate of the fluid. To simplify the problems we make the following assumptions: 1) the thermal and physical properties and the heat-transfer characteristics are constants; 2) end effects related to the finite dimensions of the wall heat exchanger can be neglected, i.e., we assume an infinite porous wall; 3) heat transfer through the skeleton of the wall is by conduction only in the direction normal to the boundary surfaces, i.e., the temperature gradient in the transverse direction can be neglected. This condition holds when the temperature difference between the central part of individual elements of the skeleton and the channel surface is small in comparison with the temperature difference between the channel surface and the moving fluid, i.e., the thermal resistance to conduction through the heat-exchanger skeleton and the thermal resistance to heat transfer to the moving fluid are controlling. This assumption will be satisfied sufficiently rigorously if the Biot number $Bi = \alpha h/\lambda$ determined in the heat-exchanger skeleton is smaller than unity. [1]. It is easy to verify that for metal heat exchangers this assumption is satisfied over reasonable variations of the heat-transfer coefficient α, the thermal conductivity λ, and a characteristic internal dimension h — the distance between centers of the openings; 4) heat conduction in the fluid can be neglected; this assumption is obvious for turbulent flow, but even for laminar flow, taking account of the fact that the thermal conductivity of a fluid is an order of magnitude smaller than that of the heat-exchanger material, heat conduction in the fluid can be neglected.

Thus, in our heat-exchanger model we assume that heat is transferred by conduction through the skeleton of the material, and by convection to the fluid. Then the system of equations for the temperature distribution (a one-dimensional problem for the material of the device) can be formulated in the following way for a 1-m^2 cross section of the heat exchanger (Fig. 1).

The amount of heat transferred to the flowing fluid in a part dx during a time $d\tau$ is [1]

$$dQ = \lambda f \frac{d^2 t_M}{dx^2} dx d\tau, \tag{1}$$

but on the other hand this heat can be determined from the heat-transfer equation

$$dQ = \alpha P (t_M - t_F) dx d\tau. \tag{2}$$

Finally, the amount of heat going into heating of the fluid is

$$dQ = \rho (1 - f) u c_p \frac{dt_F}{dx} dx d\tau. \tag{3}$$

By equating (1), (2), and (3), we obtain the following system of equations:

$$\lambda f \frac{d^2 t_M}{dx^2} = \alpha P (t_M - t_F), \tag{4}$$

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